The two-dimensional bond-diluted transverse Ising model at zero temperature

1981 J. Phys. A: Math. Gen. 14 L179

(http://iopscience.iop.org/0305-4470/14/5/014)

J. Phys. A: Math. Gen. **14** (1981) L179–L183. Printed in Great Britain

LETTER TO THE EDITOR

The two-dimensional bond-diluted transverse Ising model at zero temperature

R R dos Santos†

Department of Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK

Received 5 February 1981

Abstract. The critical behaviour of the bond-diluted two-dimensional quantum transverse Ising model is investigated by an approximate cluster decimation transformation. The dependence of the critical field on the concentration shows a discontinuity at the percolation concentration, associated with the existence of long-range order in the percolating cluster at non-zero transverse field, according to Harris' conjecture.

In this Letter we treat the diluted two-dimensional transverse Ising model at zero temperature through an approximate decimation transformation.

The effects of quenched random impurities (Brout 1959) on thermal critical behaviour of magnetic systems have been somewhat established. In addition to shifting the critical temperature, the impurities may drive the pure system to a new critical behaviour (i.e. new set of critical exponents). A condition for this crossover to occur has been given by Harris (1974a) through heuristic arguments: the critical behaviour of the pure system is stable (unstable) with respect to randomness if the specific heat exponent α is negative (positive). Also, real space renormalisation-group techniques (Niemeijer and Van Leeuwen 1976) have provided satisfactory results for critical exponents and critical curves of diluted Ising (Young and Stinchcombe 1976, Kirkpatrick 1977, Jayaprakash *et al* 1978, Yeomans and Stinchcombe 1978, 1979) and Heisenberg (Stinchcombe 1979) models. In these systems bonds (or sites) are present with probability p. As p is decreased from one, the connectivity of the lattice decreases, causing the critical temperature to decrease. Below a critical concentration p_c, the percolation concentration (Stauffer 1979), no infinite cluster of connected bonds (or sites) is formed so that long-range order is absent. It is interesting to note that when the critical behaviour of the pure system is stable with respect to randomness, the critical temperature drops continuously to zero at p_c, since the percolating cluster is chain-like.

The above picture does not necessarily hold for quantum systems undergoing a transition at zero temperature such as the transverse Ising model (TIM) (de Gennes 1963, Stinchcombe 1973), described by the Hamiltonian

$$\mathcal{H} = -\sum_i \Gamma_i \sigma_i^x - \sum_{\langle ij \rangle} J_{ij} \sigma_i^z \sigma_j^z \tag{1}$$

in which the field Γ_i and nearest-neighbour exchange coupling J_{ij} are random variables, the $\boldsymbol{\sigma}$'s are Pauli matrices and the sums run over sites on a d-dimensional lattice. In the non-random case ($\Gamma_i = \Gamma$, $J_{ij} = J$) the critical behaviour of (1) at finite temperatures

† On leave from Departamento de Física, Universidade Federal de Alagoas, Maceió, Brazil.

0305-4470/81/050179+05$01.50

(with respect to $T_c(\Gamma)$) is described by the same exponents as the d-dimensional Ising model, whereas at $T=0$ there is a transition at a critical value of Γ/J with the same exponents as the $(d+1)$-dimensional Ising model (Pfeuty 1970, Elliott and Wood 1971, Pfeuty and Elliott 1971, Young 1975, Hertz 1976, Suzuki 1976). In particular, there is a transition in the ground state of the one-dimensional TIM in which $g_c \equiv (\Gamma/J)_c = 1$ (Katsura 1962, Pfeuty 1970). In view of this, Harris (1974b) suggested the presence of a discontinuous jump in the critical curve $g_c(p)$ of the diluted TIM at zero temperature, at the percolation concentration p_c: below p_c there is no long-range order so that $g_c = 0$, whereas just above p_c the chain-like aspect of the percolating cluster suggests $g_c \geqslant 1$.

To date only a few attempts have been made to investigate such an interesting problem. Elliott and Saville (1974) and Lage (1976) used series expansions and CPA, respectively, to study the diluted TIM, but they were unable to pick out the zero-temperature behaviour. Pfeuty (1979) determined exactly the critical condition for the random transverse Ising chain at zero temperature. The difficulties with quantum renormalisation-group treatments for the non-random case make generalisations to treat the random TIM very hard, although some results in one dimension were obtained for special cases of randomness (Uzelac *et al* 1979, 1980).

A very interesting approach was used by Stinchcombe (1981a) to derive an exact decimation transformation for the pure transverse Ising chain at zero temperature from scaling properties of the two-dimensional Ising model. This treatment was then extended (Stinchcombe 1981b) to two dimensions within a Migdal–Kadanoff approximation (Migdal 1976, Kadanoff 1976). In this way a study of the diluted TIM at zero temperature was carried out exactly in one dimension and approximately in two dimensions (Stinchcombe 1981b). As the bond-shifting procedure in the Migdal–Kadanoff scheme (Migdal 1976, Kadanoff 1976) does not take into account non-commutation aspects of the Hamiltonian, it is interesting to compare the results obtained by Stinchcombe (1981b) for two dimensions with a different decimation scheme.

The non-random TIM at zero temperature in one and two dimensions was investigated by dos Santos (1980) within an approximate decimation transformation which incorporates non-commutation aspects, and can be naturally extended to dilution problems as well as to finite temperatures. In this Letter we report the extension of this method to the two-dimensional bond-diluted TIM at zero temperature.

For the pure system at zero temperature, a decimation recursion relation is constructed by examining the transformation properties of the ground state projector when every other spin on a d-dimensional lattice is summed out. The simplest approximation scheme then consists in summing out the even labelled spins in the square cluster of figure 1. If the Hamiltonians for the original and transformed systems are described by coupling constants j and $\tilde{j}'$, respectively, the renormalisation-group transformation (Wilson and Kogut 1974, Wallace and Zia 1978)

$$\tilde{j}' = \tilde{j}'(j) \tag{2}$$

is defined through

$$\langle m_1 m_3 | P'(\tilde{j}') | m_1 m_3 \rangle = \sum_{m_2 m_4} \langle m_1 m_2 m_3 m_4 | P(j) | m_1 m_2 m_3 m_4 \rangle \tag{3}$$

where P and P' are the ground state projectors for the clusters in figures 1(a) and (b) (with $j_i = j$, $i = 1, 4$), respectively. The states $|m_i\rangle$ are eigenstates of σ_i^z, and one should note that the full trace of the ground state projector is preserved under (3), in analogy

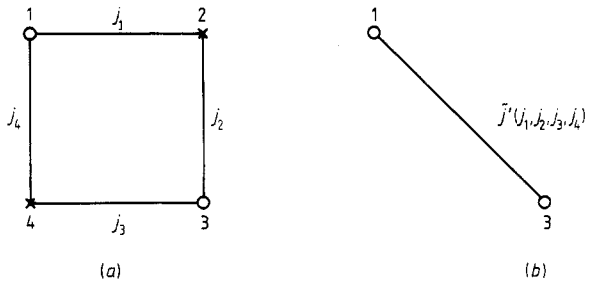

Figure 1. Clusters used in the decimation transformation for the transverse Ising model, where the couplings $j_i \equiv J_i/\Gamma$ can be either 0 or 1. The spins represented by crosses in (a) are summed out to yield the cluster in (b) where spins 1 and 3 interact via a coupling $\tilde{j}'$.

with the thermal transformations (Niemeijer and Van Leeuwen 1976). A formal discussion about the limitations of treating anisotropy exchange within diagonal transformations such as (3) is given by dos Santos (1980).

In spite of the simplicity of this sort of cluster decimation, its thermal counterpart (Kadanoff and Houghton 1975, Barber 1975) has proved very useful in elucidating several aspects of the percolation problem (Young and Stinchcombe 1975) and of diluted Ising models (Young and Stinchcombe 1976, Yeomans and Stinchcombe 1978, 1979).

In the bond-diluted TIM at zero temperature the coupling constants $j \equiv 1/g \equiv J/\Gamma$ are random variables distributed according to the probability distribution

$$P(j_i) = (1-p)\delta(j_i) + p\delta(j_i - j) \tag{4}$$

where p is the concentration of non-zero coupling constants j. Under a decimation transformation, the distribution (4) loses its binary character and, following Yeomans and Stinchcombe (1978, 1979), it can be approximated by a binary one which preserves the total probability for percolation paths as well as the first moment. In this way, the recursion relations for the concentration p and coupling constant j are given by

$$p' = 2p^2 - p^4, \tag{5}$$

$$p'j'(j) = p^4\tilde{j}'(j,j,j,j) + 4p^3(1-p)\tilde{j}'(j,j,j,0) + 2p^2(1-p)^2\tilde{j}'(j,j,0,0), \tag{6}$$

where the function $\tilde{j}'(j_1, j_2, j_3, j_4)$ is obtained from equation (3).

The RG transformation defined by (5) and (6) was solved numerically to yield the fixed points in table 1, where critical exponents obtained in the usual way (Wilson and Kogut 1974) for the non-trivial fixed points are also shown. The spurious fixed point at ($p = 0, g^* \neq 0$) is attributed to the first moment approximation, since for small p the RHS of equation (6) is dominated by the last term.

Iteration of equations (5) and (6) yields the flow diagram of figure 2 where the critical curve $g_c(p) \equiv 1/j_c(p)$ represented by bold lines is the boundary for the long-range ordered phase at $T = 0$. The logarithmic slope of the critical curve at the pure fixed point, $[(1/g_c(p))\, \mathrm{d}g_c/\mathrm{d}p]_{p=1}$, is 1.07.

Although the values obtained at the pure fixed point for the critical field $g_c = 1.55$, and for the correlation length exponent, $\nu_g = 0.49$, are not very accurate when compared with the series results $g_c = 3.04$ and $\nu_g = 0.63$ (Pfeuty and Elliott 1971), the

Table 1. Fixed points of the recursion relations (5) and (6) where $g = 1/j = \Gamma/J$, and correlation length exponents ν_g and ν_p (corresponding to quantum and percolative behaviours, respectively); ϕ is the crossover exponent.

(p^*, g^*)	ν_g	ν_p	Remarks
(0, 0); (0, ∞)	—	—	trivial
(0, 0.426)	—	—	spurious
(0.618, 0)	—	0.817	cf $p^* = 0.5$[a]; $\nu_p = 1.34$[b]
(0.618, 0.873)	0.592	0.817	$\phi \equiv \nu_g/\nu_p = 0.724$
(0.618, ∞)	—	—	trivial
(1, 0); (1, ∞)	—	—	trivial
(1, 1.55)	0.496	—	cf $g^* = 3.04$[c]; $\nu_g = 0.63$[c]

[a] Obtained from duality arguments (Sykes and Essam 1964).
[b] Obtained from series expansions (Dunn *et al* 1975).
[c] Obtained from ground state perturbation expansions (Pfeuty and Elliott 1971).

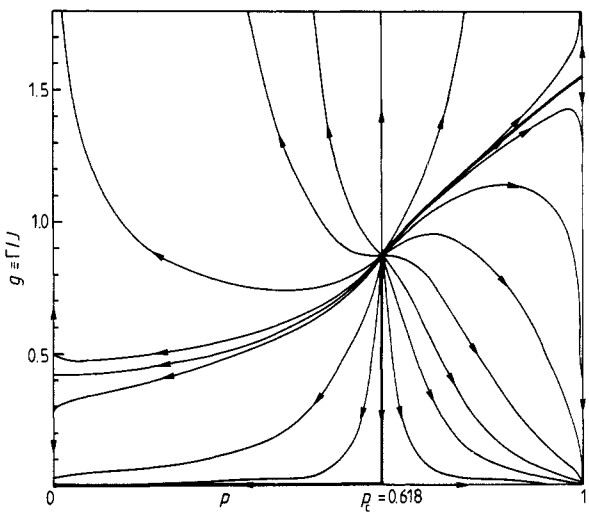

Figure 2. Flow diagram of the two-dimensional bond-diluted transverse Ising model at zero temperature, showing the fixed points and typical flow lines. The critical curve $g_c(p)$, above which there is no long-range order, is shown by bold lines.

present scheme describes the major feature expected for this problem: a discontinuous jump in the critical curve at p_c from zero to a value close to one, according to the Harris (1974b) conjecture. Moreover, the chain-like aspect of the percolating cluster suggests that $\nu_g(p = p_c) > \nu_g(p = 1)$, which is verified in our case.

As a final remark, we would like to mention that the extension of this method to obtain the phase diagram for the diluted two-dimensional TIM at any temperature is currently under investigation, allowing for site-dilution as well.

I would like to thank Dr R B Stinchcombe for many discussions and encouragement throughout this work. Financial support from the SRC is also gratefully acknowledged.

References

Barber M N 1975 *J. Phys. C: Solid State Phys.* **8** L203
Brout R 1959 *Phys. Rev.* **115** 824
Dunn A G, Essam J W and Ritchie D S 1975 *J. Phys. C: Solid State Phys.* **8** 4219
Elliott R J and Saville I D 1974 *J. Phys. C: Solid State Phys.* **7** 4293
Elliott R J and Wood C 1971 *J. Phys. C: Solid State Phys.* **4** 2359
de Gennes P G 1963 *Solid State Commun.* **1** 132
Harris A B 1974a *J. Phys. C: Solid State Phys.* **7** 1671
—— 1974b *J. Phys. C: Solid State Phys.* **7** 3082
Hertz J 1976 *Phys. Rev.* B **14** 1165
Jayaprakash C, Riedel E K and Wortis M 1978 *Phys. Rev.* B **18** 3568
Kadanoff L P 1976 *Ann. Phys., NY* **100** 559
Kadanoff L P and Houghton A 1975 *Phys. Rev.* B **11** 377
Katsura S 1962 *Phys. Rev.* **127** 1508
Kirkpatrick S 1977 *Phys. Rev.* B **15** 1533
Lage E 1976 *DPhil thesis* (Oxford)
Migdal A A 1976 *Sov. Phys.-JETP* **42** 743
Niemeijer Th and Van Leeuwen J M J 1976 in *Phase Transitions and Critical Phenomena* ed. C Domb and M S Green (New York: Academic) vol 6
Pfeuty P 1970 *Ann. Phys., NY* **57** 79
—— 1979 *Phys. Lett.* **72A** 245
Pfeuty P and Elliott R J 1971 *J. Phys. C: Solid State Phys.* **4** 2370
dos Santos R R 1980 *DPhil thesis* (Oxford)
Stauffer D 1979 *Phys. Rev.* **54C** 1
Stinchcombe R B 1973 *J. Phys. C: Solid State Phys.* **6** 2459
—— 1979 *J. Phys. C: Solid State Phys.* **12** 4533
—— 1981a to be published
—— 1981b *J. Phys. C: Solid State Phys.* to be published
Suzuki M 1976 *Prog. Theor. Phys.* **56** 1454
Sykes M F and Essam J W 1964 *J. Math. Phys.* **5** 117
Uzelac J, Jullien R and Pfeuty P 1980 *J. Phys. A: Math. Gen.* **13** 3735
Uzelac K, Penson K A, Jullien R and Pfeuty P 1979 *J. Phys. A: Math. Gen.* **12** L295
Wallace D J and Zia R K P 1978 *Rep. Prog. Phys.* **41** 1
Wilson K G and Kogut J 1974 *Phys. Rep.* **12C** 1975
Yeomans J and Stinchcombe R B 1978 *J. Phys. C: Solid State Phys.* **11** L525
—— 1979 *J. Phys. C: Solid State Phys.* **12** 347
Young A P 1975 *J. Phys. C: Solid State Phys.* **8** L309
Young A P and Stinchcombe R B 1975 *J. Phys. C: Solid State Phys.* **8** L535
—— 1976 *J. Phys. C: Solid State Phys.* **9** 4419